Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Sample Assessment Materials for first teaching September 2018

(Time: 1 hour 30 minutes) | Paper Reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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Pearson

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Answer ALL questions. Write your answers in the spaces provided.

1. Express

$$\frac{6x+4}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)

1. $\dfrac{6x+4}{9x^2+4} - \dfrac{2}{3x+1}$

↑ difference of 2 squares – $(a^2-b^2) = (a+b)(a-b)$

$= \dfrac{2\cancel{(3x+2)}}{\cancel{(3x+2)}(3x-2)} - \dfrac{2}{3x+1}$

$= \dfrac{2}{3x-2} - \dfrac{2}{3x+1}$

$= \dfrac{\cancel{6x}+2-\cancel{6x}+4}{(3x-2)(3x+1)}$

$= \dfrac{6}{(3x-2)(3x+1)}$

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2. $$f(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)} \quad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)} \quad n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3

(3)

The root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(2)

2. a) $f(x) = x^3 + 3x^2 + 4x - 12 = 0$

$x^3 + 3x^2 = 12 - 4x$

$x^2(x+3) = 4(3-x)$

$x^2 = \dfrac{4(3-x)}{x+3}$

$x = \sqrt{\dfrac{4(3-x)}{x+3}}$

b) $x_{n+1} = \sqrt{\dfrac{4(3-x_n)}{(3+x_n)}}$ $\quad n \geqslant 0$

$x_0 = 1$

$x_1 = x_{0+1} = \sqrt{\dfrac{4(3-1)}{(3+1)}} = \sqrt{\dfrac{8}{4}} = \sqrt{2} = 1.41$

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Question 2 continued

$x_2 = 1.20$

$x_3 = 1.31$

c) $f(x) = 0 \rightarrow$ root $= \alpha$

To show $\alpha = 1.272$ to 3 d.p.

$f(1.2715) = 1.2715^3 + 3(1.2715)^2 + 4(1.2715) - 12 = -0.0082$

$f(1.2725) = 0.0083$

↳ given that $f(x)$ is continuous between 1.2715 & 1.2725,

because there's a sign change → root α lies between 1.2715 & 1.2725

$\therefore \alpha = 1.272$ to 3 d.p.

3.

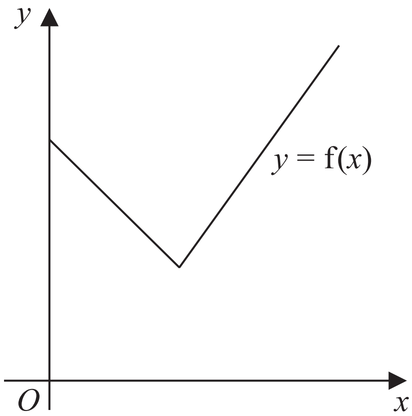


Figure 1

Figure 1 shows a sketch of part of the graph $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2|3 - x| + 5 \qquad x \geqslant 0$$

(a) Solve the equation

$$\mathrm{f}(x) = \frac{1}{2}x + 30$$

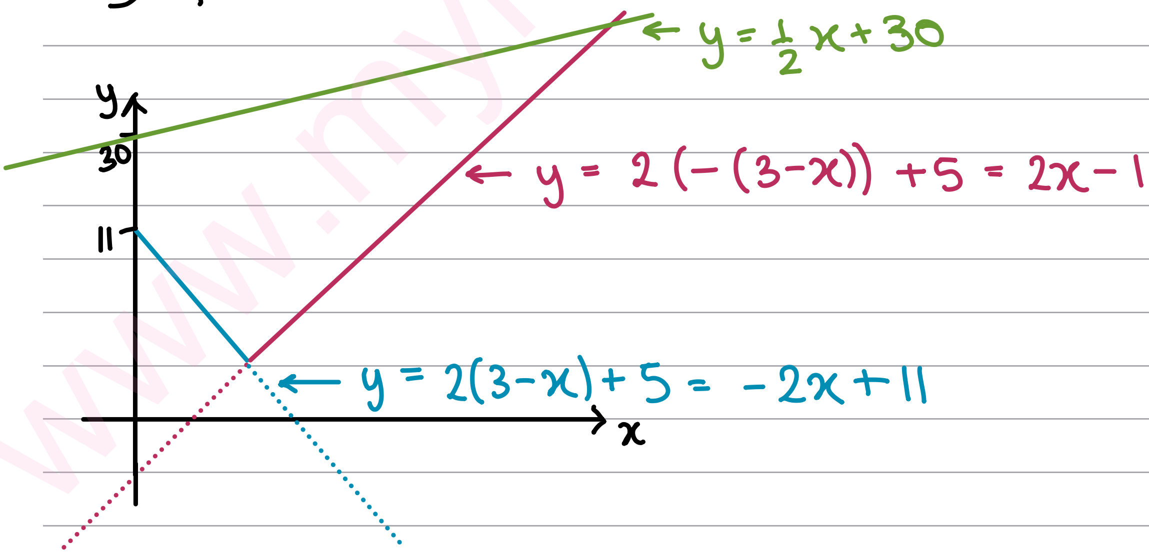
(3)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has two distinct roots,

(b) state the set of possible values for k.

(2)

3. (a) $f(x) = 2|3-x| + 5 \qquad x \geq 0$



∴ as we can see from the diagram

$y = \frac{1}{2}x + 30$ intersects with $y = 2x - 1$ part of $f(x)$

Question 3 continued

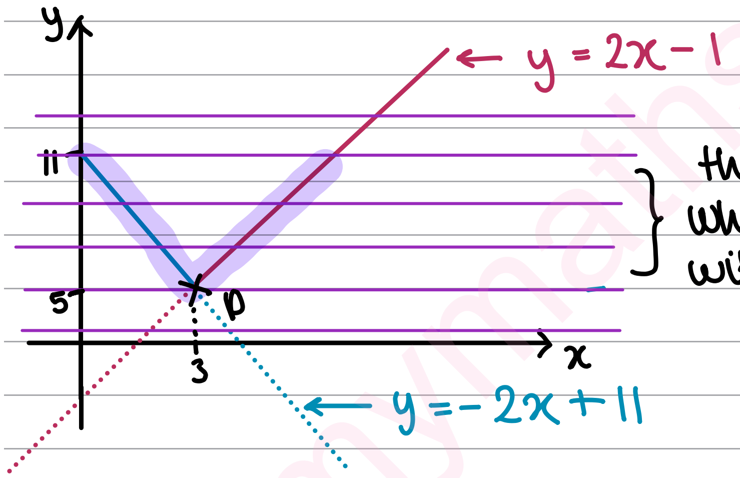
$$2x-1 = \frac{x}{2} + 30$$

$$4x - 2 = x + 60$$

$$3x = 62$$

$$x = \frac{62}{3}$$

b) given $f(x) = k$ has 2 distinct roots



there are 2 distinct roots when there are 2 intersections with $f(x)$

P is the point of intersection between both parts of $f(x)$:

$$2x-1 = -2x+11$$

$$4x = 12$$

$$x = 3 \rightarrow P = (3, 5)$$

Q3

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Question 3 continued

$\therefore$ when $k < 5 \rightarrow$ there are 0 intersections with $f(x)$

when $k = 5 \rightarrow$ there is 1 intersection (at P)

when $k > 11 \rightarrow$ there is only 1 intersection with $y = 2x - 1$

$\therefore$ for 2 intersections

$$5 < k \leq 11$$

Q3

(Total for Question 3 is 5 marks)

4. (i) Find

$$\int_{5}^{13} \frac{1}{(2x-1)}\,dx$$

writing your answer in its simplest form.

(4)

(ii) Use integration to find the exact value of

$$\int_{0}^{\frac{\pi}{2}} \sin 2x + \sec \frac{1}{3}x \tan \frac{1}{3}x\,dx$$

(3)

4. (i) $\displaystyle\int_{5}^{13} \frac{1}{2x-1}\,dx$

$= \left[\frac{1}{2}\ln(2x-1)\right]_{5}^{13}$

$= \frac{1}{2}\left(\ln(25) - \ln(9)\right)$

$= \frac{1}{2}\ln\left(\frac{25}{9}\right)$

$= \ln\left(\left(\frac{25}{9}\right)^{\frac{1}{2}}\right)$

$= \ln\left(\frac{5}{3}\right)$

LOG RULES

$a\log_b(c) = \log_b(c^a)$

$\log_a b - \log_a c = \log_a\left(\frac{b}{c}\right)$

Question 4 continued

(ii) $\int_0^{\frac{\pi}{2}} \sin(2x) + \sec\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right) \, dx$

$= \left[-\frac{1}{2}\cos(2x) + 3\sec\left(\frac{x}{3}\right)\right]_0^{\frac{\pi}{2}}$

$= -\frac{\cos(\pi)}{2} + 3\sec\left(\frac{\pi}{6}\right) + \frac{\cos(0)}{2} - 3\sec(0)$

$= \frac{1}{2} + 2\sqrt{3} + \frac{1}{2} - 3$

$= 2\sqrt{3} - 2$

5. Given that

$$y = \frac{5x^2 - 10x + 9}{(x-1)^2} \qquad x \neq 1$$

show that $\frac{dy}{dx} = \frac{k}{(x-1)^3}$, where k is a constant to be found.

(6)

5. $y = \frac{5x^2 - 10x + 9}{(x-1)^2}$ $\qquad x \neq -1$

Quotient rule for differentiating : $y = \frac{u}{v} \rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$u = 5x^2 - 10x + 9 \qquad \frac{du}{dx} = 10x - 10$

$v = (x-1)^2 \qquad \frac{dv}{dx} = 2x - 2$

$$\frac{dy}{dx} = \frac{((x-1)^2)(10x-10) - (5x^2 - 10x + 9)(2x-2)}{((x-1)^2)^2}$$

$$= \frac{10(x-1)^3 - 2(5x^2 - 10x + 9)(x-1)}{(x-1)^4}$$

$$= \frac{10(x-1)^2 - 2(5x^2 - 10x + 9)}{(x-1)^3}$$

$$= \frac{\cancel{10x^2} - \cancel{20x} + 10 - \cancel{10x^2} + \cancel{20x} - 18}{(x-1)^3}$$

$$= \frac{-8}{(x-1)^3} \qquad k = -8$$

6. The functions f and g are defined by

$$f : x \mapsto e^x + 2 \qquad x \in \mathbb{R}$$

$$g : x \mapsto \ln x \qquad x > 0$$

(a) State the range of f. **(1)**

(b) Find fg(x), giving your answer in its simplest form. **(2)**

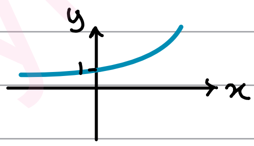
(c) Find the exact value of x for which $f(2x + 3) = 6$ **(4)**

(d) Find f^{-1} stating its domain. **(3)**

(e) On the same axes sketch the curves with equation $y = f(x)$ and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes. **(4)**

6.a) $f : x \rightarrow e^x + 2 \qquad x \in \mathbb{R}$

range of e^x :



← as seen from the graph

range of $e^x > 0$

$\therefore \; f(x) > 2$

b) $f(g(x)) = f(\ln(x))$

$= e^{\ln(x)} + 2$

$= x + 2$

c) $f(2x+3) = 6$

$e^{2x+3} + 2 = 6$

$e^{2x+3} = 4$

$2x+3 = \ln(4)$

$x = \dfrac{\ln(4) - 3}{2}$

d) to find $f^{-1}(x)$: $f(x) = e^{x} + 2 \quad x \in \mathbb{R}$

① write the function using a "y" and set equal to "x" $x = e^{y} + 2$

$e^{y} = x - 2$

② rearrange to make y the subject : $y = \ln(x-2)$

③ replace y with $f^{-1}(x)$: $\therefore f^{-1}(x) = \ln(x-2)$

Because we are told to find $f^{-1}(x)$, we must also state the domain of the inverse function :

↑ domain refers to the set of values we are allowed to plug into our function

domain of inverse function = range of function

↑ range refers to all possible values of a function

$\therefore$ domain of $f^{-1}(x)$ = range of $f(x)$

$\therefore f^{-1}(x) = \ln(x-2) \qquad x > 2$

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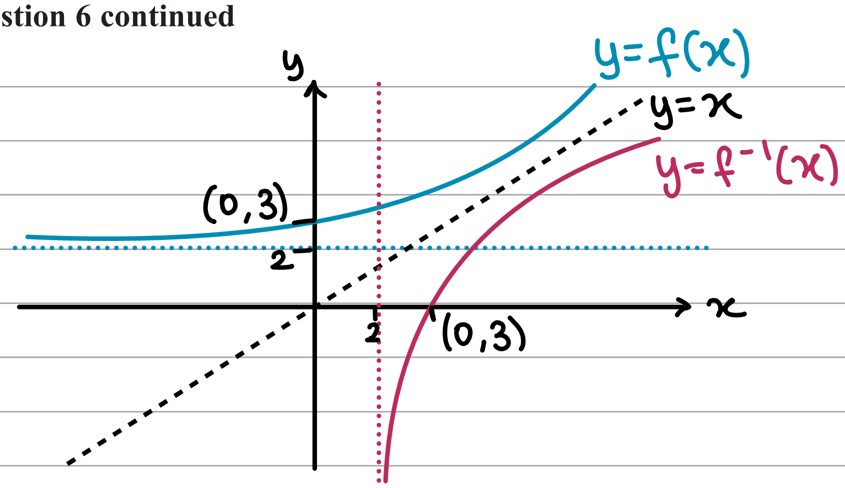
Question 6 continued

e)



7. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

7. a) $x = (4y - \sin(2y))^2$

$P: \left(p, \frac{\pi}{2}\right)$

$$p = \left(4\left(\frac{\pi}{2}\right) - \sin\left(2\left(\frac{\pi}{2}\right)\right)\right)^2$$

$$p = (2\pi)^2 = 4\pi^2$$

b) gradient of tangent at P $= \frac{dy}{dx}(P)$

$x = (4y - \sin(2y))^2$

CHAIN RULE : $\frac{dy}{dx} = \frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{dx}$

$v = 4y - \sin(2y)$ $\qquad \frac{dv}{dy} = 4 - 2\cos(2y)$

$x = v^2$ $\qquad \frac{dx}{dv} = 2v$

Question 7 continued

$$\frac{dx}{dy} = \frac{dx}{dv} \times \frac{dv}{dy}$$

$$= 2v \times 4 - 2\cos(2y)$$

$$= 2(4y - \sin(2y))(4 - 2\cos(2y))$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\left(dx/dy\right)}}$$

$$\frac{dy}{dx} = \frac{1}{2(4y - \sin(2y))(4 - 2\cos(2y))}$$

$\therefore$ gradient at P $\left(4\pi^2, \frac{\pi}{2}\right)$

$$\frac{dy}{dx}(P) = \frac{1}{2\left(4\left(\frac{\pi}{2}\right) - \sin\left(2\left(\frac{\pi}{2}\right)\right)\right)\left(4 - 2\cos\left(2\left(\frac{\pi}{2}\right)\right)\right)}$$

$$= \frac{1}{2(2\pi)(6)}$$

$$= \frac{1}{24\pi}$$

Equation of line : $y - y_1 = m(x - x_1)$ coordinates of known point on line; gradient

$\therefore$ equation of tangent to P : $y - \frac{\pi}{2} = \frac{1}{24\pi}(x - 4\pi^2)$

Question 7 continued

tangent cuts y-axis when $x = 0$ at $A \rightarrow (0, a)$

$$a - \frac{\pi}{2} = \frac{1}{24\pi}(0 - 4\pi^2)$$

$$a - \frac{\pi}{2} = -\frac{\pi}{6}$$

$$a = \frac{\pi}{3}$$

$$\therefore \quad A = \left(0, \frac{\pi}{3}\right)$$

8. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m\log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

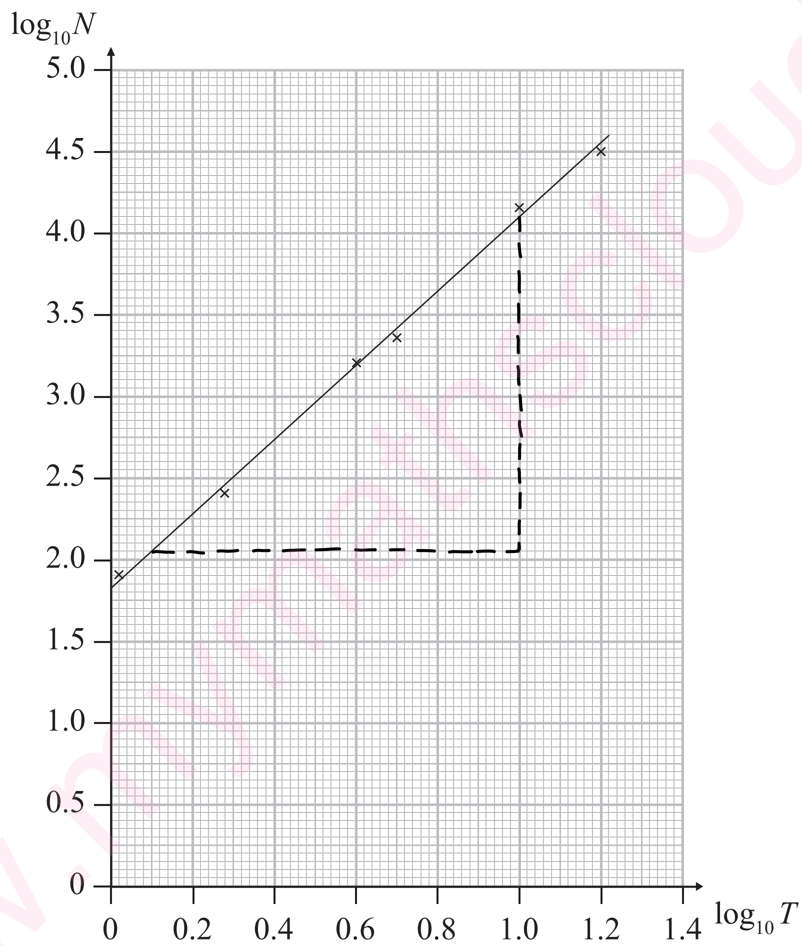


Figure 2

Figure 2 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) With reference to the model, interpret the value of the constant a.

(1)

Question 8 continued

8. a) $N = aT^b$

$\log_{10}(N) = \log_{10}(aT^b)$

$\log_{10}(N) = \log_{10}(a) + \log_{10}(T^b)$

$\log_{10}(N) = \log_{10}(a) + b\log_{10}(T)$

$y = c + m\log_{10}(T)$

LOG RULES

$a\log_b(c) = \log_b(c^a)$

$\log_a b + \log_a c = \log_a(bc)$

$\log_a b = c \rightarrow a^c = b$

$c = \log_{10}(a)$ $\quad$ $m = b$

b) y-intercept $= c = 1.85 = \log_{10}(a)$

$a = 10^{1.85} = 70.8$

gradient $= m = \frac{\Delta y}{\Delta x} = \frac{4.1 - 2.05}{1 - 0.1} = \frac{41}{18} = b$

choose 2 random points

$N_3 = aT^b = 70.8 \times (3)^{\frac{41}{18}}$

$= 865$ microbes

c) $N_1 = aT^b = a(1)^b = a$ $\therefore$ a is number of microbes after 1 day

9. (a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A} \qquad A \neq \frac{(2n+1)\pi}{4} \qquad n \in \mathbb{Z}$$

(5)

(b) Hence solve, for $0 \leqslant \theta < 2\pi$

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}$$

Give your answers to 3 decimal places.

(4)

9.a) LHS : $\sec(2A) + \tan(2A)$ RHS : $\dfrac{\cos(A) + \sin(A)}{\cos(A) - \sin(A)}$

$$= \frac{1}{\cos(2A)} + \frac{\sin(2A)}{\cos(2A)}$$

$$= \frac{\sin(2A) + 1}{\cos(2A)}$$

$\boxed{\sin(2A) = 2\sin(A)\cos(A)}$

$\boxed{\cos^2(A) + \sin^2(A) = 1}$

$\boxed{\cos(2A) = \cos^2(A) - \sin^2(A)}$

$$= \frac{2\sin(A)\cos(A) + 1}{\cos^2(A) - \sin^2(A)}$$

$$= \frac{2\sin(A)\cos(A) + 1}{(\cos(A) + \sin(A))(\cos(A) - \sin(A))}$$

$$= \frac{2\sin(A)\cos(A) + \cos^2(A) + \sin^2(A)}{(\cos(A) + \sin(A))(\cos(A) - \sin(A))}$$

$$= \frac{(\cos(A) + \sin(A))^{\cancel{2}}}{\cancel{(\cos(A) + \sin(A))}(\cos(A) - \sin(A))}$$

$$= \frac{\cos(A) + \sin(A)}{\cos(A) - \sin(A)} = \text{RHS}$$

Question 9 continued

b) $\sec(2\theta) + \tan(2\theta) = \frac{1}{2}$ $\qquad 0 \leqslant \theta < 2\pi$

using identity from (a)

$$\frac{\cos(\theta) + \sin(\theta)}{\cos(\theta) - \sin(\theta)} = \frac{1}{2}$$

$$2\cos(\theta) + 2\sin(\theta) = \cos(\theta) - \sin(\theta)$$

$$3\sin(\theta) = -\cos(\theta)$$

$$\tan(\theta) = -\frac{1}{3}$$

$$\theta = 2.820 \,,\; 5.961$$

10. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t}$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(b) show that the total amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

(2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(c) Show that $T = a \ln\left(b + \dfrac{b}{e}\right)$, where a and b are integers to be determined.

(4)

10. a) $x = De^{-0.2t}$

$D = 15$

when $t = 4$ $\rightarrow$ $x = 15 \times e^{-0.2(4)}$

$= 6.740$ mg

b) when $x = 7$ $\rightarrow$ $x_1 = 15 \times e^{-0.2(7)}$ 7 hours after 1st dose

$+$

$x_2 = 15 \times e^{-0.2(2)}$ 2 hours after 2nd dose

$= 13.754$ mg

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Question 10 continued

c) T hours after 2nd dose (= T+5 after 1st dose) : $7.5 = x_1 + x_2$

$$= 15e^{-0.2(T+5)} + 15e^{-0.2(T)}$$

$$7.5 = 15e^{-0.2T-1} + 15e^{-0.2T}$$

$$\frac{1}{2} = e^{-0.2T}(e^{-1}+1)$$

$$e^{-0.2T} = \frac{1}{2(e^{-1}+1)}$$

$$T = \frac{-1}{0.2} \times \ln\left(\frac{1}{2(\frac{1}{e}+1)}\right)$$

$$T = -5 \times \ln\left(\frac{e}{2+2e}\right)$$

$$T = 5\ln\left(\left(\frac{e}{2+2e}\right)^{-1}\right)$$

$$T = 5\ln\left(\frac{2+2e}{e}\right) = 5\ln\left(\frac{2}{e}+2\right)$$

$a = 5 \quad b = 2$